

We consider the unitary operator corresponds to 1-dimensional split-step quantum walks (SSQWs) introduced by Kitagawa. This operator possesses chiral symmetry. Based on the theory of chiral symmetric quantum walks, supercharge correspond to SSQW is naturally introduced. If supercharge satisfies a Fredholm condition, a Witten index coincides with a Fredholm index and it gives a lower bound of the sum of dimensions of eigenspaces at 1 and -1. In this talk, we study the Witten index of supercharge under the non-Fredholm condition. We make use of the spectral shift function induced by the 4th order difference operator with a rank 1 perturbation on the discrete half-line. Under the two-phase condition, the Witten index only depends on two side limits of parameters and half-integers appear. This talk is based on joint works with Yasumichi Matsuzawa (Shinshu univ.), Akito Suzuki (Shinshu univ.), Yohei Tanaka (Shinshu univ.), and Noriaki Teranishi (Hokkaido univ.).